Broward County Schools
AP Physics 1
Review

AP Physics 1 Review

The Basics of the Exam

Section	Type of Questions	Number of Questions	Weighting	Timing
I	Multiple-choice questions	40	50%	80 minutes
II	Free-response questions	4	50%	100 minutes
	Question 1: Mathematical Routines			
	Question 2: Translation Between Representations			
	Question 3: Experimental Design and Analysis			
	Question 4: Qualitative/Quantitative Translation			

Search for "**AP Physics 1 CED**" to find the AP Physics 1 Course and Exam Description pdf. You will find sample questions of the Free Response question types with rubrics. These are REALLY useful for understanding the types of questions you will be asked!

Exam Weighting for the Multiple-Choice Section of the AP Exam

Units of Instruction	Exam Weighting
Unit 1: Kinematics	10–15%
Unit 2: Force and Translational Dynamics	18–23%
Unit 3: Work, Energy, and Power	18–23%
Unit 4: Linear Momentum	10–15%
Unit 5: Torque and Rotational Dynamics	10–15%
Unit 6: Energy and Momentum of Rotating Systems	5–8%
Unit 7: Oscillations	5–8%
Unit 8: Fluids	10–15%

Important info:

- There will always be four answer choices.
- No penalty for guessing. Eliminate one or two choices and then take a shot.
- You can use the reference sheet and calculator for both parts of the exam.
- Time is important, so do not spend too much time on any one question.
- All MC questions are worth the same number of points.
- Use $g = 10\ m/s^2$ to make calculations easier and the numbers work out better.
- Don't go crazy with sig figs and keep in mind precision/error on lab questions.

Free-Response Questions

The free-response section of the AP Physics 1 Exam consists of four question types listed below in the order they will appear on the exam.

Mathematical Routines (MR)

Skills: 1.A 1.C 2.A 2.B 3.B 3.C

10 points; suggested time: 20-25 minutes

The Mathematical Routines (MR) question assesses students' ability to use mathematics to analyze a scenario and make predictions about that scenario. Students will be expected to symbolically derive relationships between variables, as well as calculate numerical values. Students will be expected to create and use representations that describe the scenario, either to help guide the mathematical analysis (such as drawing a free-body diagram) or that are applicable to the scenario (such as sketching velocity as a function of time).

For AP Physics 1 and AP Physics 2, the MR question will ask students to make a claim or prediction about the scenario and use appropriate physics concepts and principles to support and justify that claim. The justification is expected to be a logical and sequential application of physics concepts that demonstrates a student's ability to connect multiple concepts to each other.

Translation Between Representations (TBR)

Skills: 1.A 1.C 2.A 2.D 3.B 3.C

12 points; suggested time: 25-30 minutes

The Translation Between Representations (TBR) question assesses students' ability to connect different representations of a scenario. Students will be expected to create a visual representation that describes a given scenario. Students will derive equations that are mathematically relevant to the scenario. Students will draw graphs that relate quantities within the scenario. Finally, students will be asked to do any one of the following:

- Justify why their answers to any two of the previous parts do/do not agree with each other.
- Use their representations, mathematical analysis, or graph to make a prediction about another situation and justify their prediction using that reasoning or analysis.
- Use their representations, mathematical analysis, or graph to make a prediction about how those representations would change if properties of the scenario were altered and justify that claim using consistent reasoning or analysis.

Experimental Design and Analysis (LAB)

Skills: 1.B 2.B 2.D 3.A

10 points; suggested time: 25-30 minutes

The Experimental Design and Analysis (LAB) question assesses students' ability to create scientific procedures that can be used with appropriate data analysis techniques to determine the answer to given questions. The LAB question can roughly be divided into two sections: Design and Analysis. In the Design portion of the LAB question, students will be asked to develop a method by which a question about a given physical scenario could be answered. The experimental procedure is expected to be scientifically sound: vary a single parameter, and measure how that change affects a single characteristic. Methods must be able to be performed in a typical high school laboratory. Measurements must be made with realistically obtainable equipment or sensors. Students will be expected to describe a method by which the collected data could be analyzed in order to answer the posed question, by either graphical or comparative analyses.

Students will then be given experimental data collected in order to answer a similar, but not identical, question to what was asked in the Design portion of the question. Students will be asked to use the data provided to create and plot a graph that can be analyzed to determine the answer to the given question. For instance, the slope or intercepts of the line may be used to determine a physical quantity or perhaps the nature of the slope would answer the posed question.

Qualitative/Quantitative Translation (QQT)

Skills: 2.A 2.D 3.B 3.C

8 points; suggested time: 15-20 minutes

The Qualitative/Quantitative Translation (QQT) question assesses students' ability to connect the nature of the scenario, the physical laws that govern the scenario, and mathematical representations of that scenario to each other. Students will be asked to make and justify a claim about a given scenario, as well as derive an equation related to that scenario. Finally, students will be asked to do any one of the following:

- Justify why their answers to any of the previous parts do/do not agree with each other.
- Use their representations or mathematical analysis to make a prediction about another situation and justify their prediction using that reasoning or analysis.
- Use their representations and mathematical analysis to make a prediction about how those representations would change if properties of the scenario were altered and justify that claim using consistent reasoning or analysis.

While students may not be directly assessed on their ability to create diagrams or other representations of the system to answer the QQT, those skills may still help students to answer the QQT. For instance, some students may find that drawing a free-body diagram is useful when determining the acceleration of a system. However, the student will earn points for the explanation and conclusions that diagram indicates (or perhaps the derivation that results from the diagram), rather than for creating the diagram itself.

Graphing and Labs

For the experimental design question, do all of these!!!

- Use a step-by-step procedure to explain how you will get data
- Explain exactly which variables you will measure
- Be specific about what measuring tools you will use (ruler, stopwatch, photogates, video, …)
- You have an "action step" to explain that something is happening, and you are measuring it
- Only change one variable from one trial to the next
- Always mention that you will repeat each trial multiple times and average results to get each data point to reduce lab error

What to Use to Measure Experimental Quantities

Distance – meter stick or tape measure
Time – stopwatch or video analysis
Mass – balance or scales
Force – force sensor or spring scale
Velocity – distance & time or motion detector
Acceleration – distance & time or motion detector
Period –stopwatch (10 oscillations and ÷ 10)

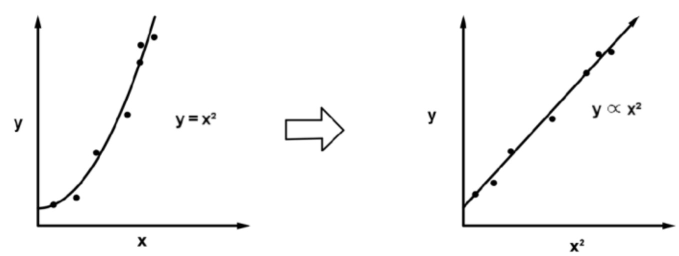
State which specific variables to measure to get the necessary data and which exact instruments to make the measurements. State which variable to change each trial, and that those trials should be repeated multiple times with the results averaged.

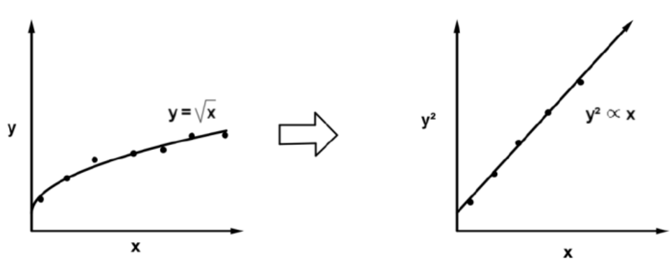
Linear or Nonlinear Data?

Sometimes one variable will directly affect another, and the data on your graph will be linear. However, many times you will not be able to fit a line to your data very well. While there are an infinite number of nonlinear relationships between variables, you will find that the vast majority of graphs will fall into one of these three types:

Quadratic

If the dependent variable (*y*) increases at a much faster rate than the independent variable (*x*), the relationship between is likely to be *quadratic*. You will notice the data tends to curve towards the *y*-axis. If your data looks like the graph on the right, you can likely make the data line up by squaring the independent variable (*x*). A second graph of the dependent variable on the *y*-axis and the independent variable squared on the *x*-axis is likely to be linear.

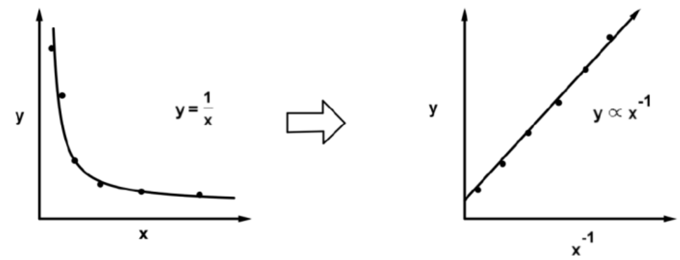




Square Root

If the dependent variable (*y*) increases at a much slower rate than the independent variable (*x*), the relationship between them is likely to be a *square root*. The points appear to curve towards the *x*-axis. If your data looks like the graph on the far left, you can probably make the data line up by squaring the dependent variable (*y*). While it's also possible to square root the independent variable, this method is harder and not recommended. A second graph of the dependent variable squared on the *y*-axis and the independent variable on the *x*-axis is likely to be linear.

Inverse

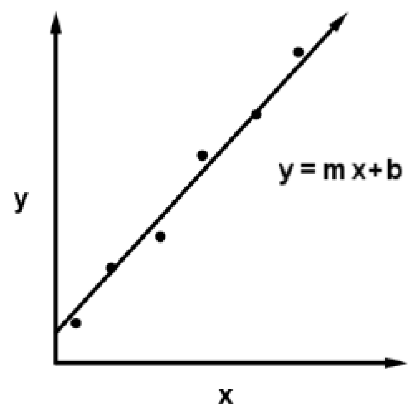


The last type happens when the dependent variable (y) decreases as the independent variable (x) increases. This is most likely an *inverse* relationship. If your data looks like the very distinctive graph on the right, you can likely make the data line up by graphing the inverse, or reciprocal, of the independent variable (x). A second graph of the dependent variable on the y-axis and the reciprocal of the independent variable on the x-axis is likely to be linear. Again, while it's also possible to inverse the dependent variable instead, it's not the best approach.

Best Fits Line (aka Regression Line)

If your data is linear, or if you have managed to make a second graph come out linear using one of the operations above, then you should use a best fits line. When drawing a best fits line try to draw it right down the middle of the points. A common mistake is to use two points – often one on either side of the data – to use for your line. This should not be done.

Finding the Equation of the Best Fits Line



After drawing the best fits line, you need to find the equation of the line. At this point, no data points should be used to do any calculations. Your best fits line is like an average of your data and is better than any data points. Find two points on your line, as far apart as you can, and the formula, $\boldsymbol{m} = (\boldsymbol{y_2}-\boldsymbol{y_1})/(\boldsymbol{x_2}-\boldsymbol{x_1})$, to find the slope of the line.
Use the slope, with one point, and the formula, $\boldsymbol{y} = \boldsymbol{mx} + \boldsymbol{b}$, to find the y-intercept. A very common mistake is failure to account for units and scientific notation when doing slope and y-intercept calculations. After finding the slope and y-intercept of your graph, it is important to write an equation that best represents your data. While it is OK to use variables like x and y when doing your work, it is very unlikely that those were the variables you used in the experiment. It is therefore very important that you replace x and y with the actual variables that you graphed on the x and y axes to get a linear graph. You replace x and y in the equation with the actual variables from the axes of your linear graph.

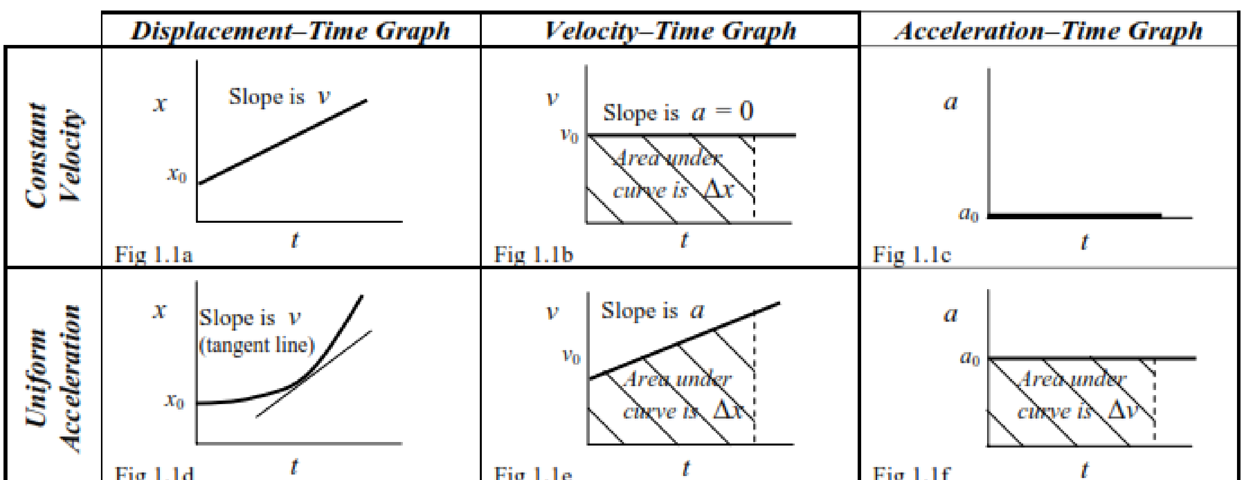
Slopes, *y*-Intercepts, Areas under a Line and Equations Mean Something in Physics

If this were a math class, you'd probably be done after finding the correct equation. But this is physics! The slope, the y-intercept, the area between the line and an axis, and the equation all have physical meaning. By matching up your equations to actual physics formulas, you can figure out the physical meanings of these values – known as experimental values – and how they can often match up to theoretical values like known scientific constants. This will allow you to have an indication of how good the experiment was, or how carefully you measured and analyzed your data, or both.

- On graphing lab questions, manipulate the data given using a physical law, if possible
- Always label axes with variables and units
- When finding slope from a best fits line, do NOT use data points
- The slope can be used to find physical quantities, if you know the right formula

Unit 1: Kinematics

- Position-Time Graphs – Represent 1-D motion with *velocity as slope*
- Velocity-Time Graphs – Represent 1-D motion with *acceleration as slope* and *displacement as area under the curve* (between the curve and the axis)
- Displacement (meters) $\quad \Delta x = x_f - x_i$
- Average Velocity (meters/second) $\quad v = \frac{\Delta x}{\Delta t}$
- Acceleration (meters/second^2) $\quad a = \frac{\Delta v}{\Delta t}$
- Problems in Algebra based physics will have constant acceleration, or should be broken into a series of constant acceleration parts
- The important thing about these equations is defining your given information correctly, and units help quite a bit in this regard
    - $v_f = v_i + at$
    - $v_f^2 = v_i^2 + 2a\Delta x$
    - $\Delta x = v_i t + \frac{1}{2}at^2$
    - $\Delta x = \frac{1}{2}(v_i + v_f)t$
- Free Fall
    - When an object has only gravity acting on it (projectiles with no air resistance, orbiting objects, etc)
    - Use kinematic equations with $a_g = -10$ m/s^2 and up as positive, down as negative
    - Vertical velocity at top of flight is zero for an instant
    - Symmetry of flight is a beautiful and important part, as the speed at every height on the way up will be the speed at the same height on the way down (no air resistance)
- If an object's velocity and acceleration are in same direction, regardless of sign, object is speeding up. If velocity and acceleration are in opposite directions, object is slowing down.

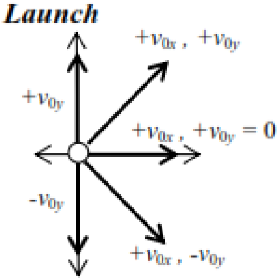
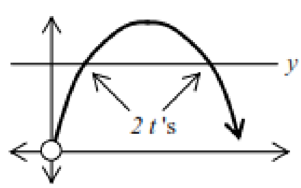
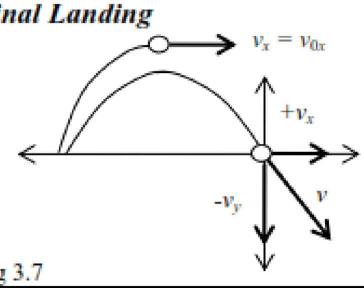


Slopes of Curves are an important analytical tool used in physics. Any equation that can be manipulated into the format $y = mx + b$ can be represented and analyzed graphically. As an example: $v = v_0 + at$ can be rearranged slightly into $v = at + v_0$. Compare this equation to the equation of a line. It is apparent that a is the slope and that v_0 is the y-intercept (Fig 1.1b and Fig 1.1e). What equation generates velocity in Fig. 1.1a and Fig 1.1d?

Area Under a Curve is another important graphical tool. Multiply the y-axis (height) by the x-axis (base) and determine if this matches any known equations. For example: Figures 1.1b and 1.1e are velocity-time plots. Simply multiply $v \times t$. This is a rearranged form of the equation $v = x/t$. The form obtained from the graph is $x = vt$, which means that displacement is the area under the velocity-time plot.

Projectiles

- Draw a quick sketch
- This motion is parabolic, and the path is referred to as the trajectory
- Horizontal (forward) velocity is CONSTANT! (if there's no air resistance)
- Vertical and horizontal motion is INDEPENDENT!
- Horizontal Launch
  - Initial velocity in vertical direction is ZERO m/s!
  - Make two columns of given info (displacement, velocity, acc)
  - Vertical ($\boldsymbol{\Delta y} \quad \boldsymbol{v_{yi} = 0} \quad \boldsymbol{a_y = -10\ m/s^2}$) and Horizontal ($\boldsymbol{\Delta x} \quad \boldsymbol{v_{xi} = v_i} \quad \boldsymbol{a_y = 0}$)
  - Use the same kinematic equations from above, with the given info
  - Use the column with more info to find **time in the air**
  - Use time in the air with info from other column to find answer
- Vertical Launch
  - Given launch velocity (v) and angle with horiz (θ), $\boldsymbol{v_{yi} = v \sin\theta}$ and $\boldsymbol{v_{xi} = v \cos\theta}$
  - If it lands on level ground (same height as launch), $\boldsymbol{v_{yf} = -v \sin\theta}$
  - Velocity at top (apex) is NOT zero, but $\boldsymbol{v_{y,top} = 0}$ and $\boldsymbol{v_{x,top} = v \cos\theta}$ (max height)
  - Most of the time, find the **time in the air**, and use that to help find the answer

Initial Launch	***During the problem***	***Final Landing***
 Fig 3.5	(At the top $v_x = v_{ox}$ and $v_y = 0$)  Fig 3.6	 Fig 3.7
Initial displacement $x_o = 0$ $y_o = 0$ Falling bodies: $\theta = \pm 90^o$ $v_{0x} = v_0 \cos 90^o = 0$ $v_{0y} = v_o \sin 90^o = v_o$ Horizontal launch: $\theta = 0^o$ $v_{0x} = v_0 \cos 0^o = v_0$ $v_{0y} = v_0 \sin 0^o = 0$ 1st quadrant launch: $+\theta$ $v_{0x} = v_0 \cos\theta$ will be + $v_{0y} = v_0 \sin\theta$ will be + 4th quadrant launch $-\theta$ $v_{ox} = v_o \cos\theta$ will be + $v_{oy} = v_o \sin\theta$ will be –	If it lands at the same height as it started ($y = y_0$), then $t_{up} = t_{down}$. There are two t's for every y. The shorter t is for the upward trip. The longer t is for the downward trip. Solve for maximum height two ways 1. From ground up where $v_y = 0$. $v_y^2 = v_{0y}^2 + 2g(y - y_0)$ 2. Or the easy way. Start at the top and pretend it is a falling body. v_x doesn't matter since time is controlled by the y-direction. And at the top v_{oy} is zero. However, this solves for half of the total flight. $y = \frac{1}{2}gt^2$ ***<u>Must double time!</u>***	$+x$ Always $-y$ Lands lower than y_0 $y = 0$ Lands same height as y_0 $+y$ Lands higher than y_0 $a_x = 0$ No a in the x-direction $v_x = v_{0x}$ What is it doing at the end of the problem in the y-direction? $\boxed{\pm v_y}$ It is usually moving downward at the end of the problem. So v_y is ***<u>usually negative</u>*** The final v must be resolved. $v = \sqrt{v_x^2 + v_y^2}$ If $y = y_0$ $v_y = -v_{0y}$ & $v = v_0$

Unit 2: Force and Translational Dynamics

- Newton's 1st Law – Law of Inertia
  - Inertia is the tendency of an object to remain in its state of motion
  - Objects in equilibrium remain that way, unless acting on by an unbalanced force
- Newton's 2nd Law – $\boldsymbol{\Sigma F = ma}$
  - Acceleration is directly related to force, and inversely related to mass
- Newton's 3rd Law – Interaction Pairs
  - When an object exerts force on a second object, the second object simultaneously exerts an equal force on the first, in the opposite direction
  - Two forces are involved, acting on separate objects – never on a single one
- Always, always draw a FBD, to scale if possible. Label forces with distinguishing names.
- Use your FBD to write a net force equation, or two for 2-D problems
  - If it's Equilibrium, set $\boldsymbol{\Sigma F = 0}$
  - If it's Accelerated motion, set $\boldsymbol{\Sigma F = ma}$
- When making net force equations, do forward forces minus backward forces, <u>no matter the direction</u>
- Frame of reference is important:
  - Most questions are done with vertical and horizontal components
  - But on ramps, it makes sense to do parallel and perpendicular to the ramp
  - On circular motion problems, radial and tangential makes more sense
  - In general, put direction of travel, or acceleration, on one axis to make math easier
- Weight or Force of Gravity (Newtons)
  - Draw and label this first
  - $\boldsymbol{F_g = mg}$
- Normal Force (Newtons)
  - *Sometimes* it's opposite the weight, but it's *always* perpendicular to the surface
- Friction (Newtons)
  - Opposes motion
  - Coefficient of friction is unit-less and between 0 and 1
  - Static friction can be anything from zero to $\boldsymbol{F_{s,max} = \mu_s F_N}$, but is never greater than forward (applied) force
  - Kinetic friction ($\boldsymbol{F_k = \mu_k F_N}$) can be greater than forward (applied) force, in which case the object is slowing down
- Systems Approach
  - On certain questions, with multiple objects in motion together, like blocks in contact or Atwood's machine, it makes sense to think of the total net force (driving force) divided by total mass to get acceleration
- Centripetal Force (Newtons)
  - Left on their own, objects in motion will move in a straight line, as described by Newton's 1st Law. So, an object moving in a circle must have a force keeping it on its circular path. Whatever that force is, it's directed toward the center of the circle and is referred to as the centripetal force.
  - Many forces can play the role of the centripetal force: Tension, Gravity, Normal, Friction, Magnetic, or any combination
  - The <u>net force</u> directed towards the center is the centripetal force, $\boldsymbol{F_c = \frac{mv^2}{r}}$
  - If centripetal force is removed, object moves in a straight line, tangent to the circle

- Kepler's Laws of Planetary Motion
  - 1st Law – All planets move in elliptical orbits with the Sun at one focus of ellipse
    - While elliptical, these orbits are very nearly circular (doesn't effect seasons!)
    - Exact right speed and distance are needed for a circular orbit (man-made objects)
  - 2nd Law – A line connecting the planet to the Sun sweeps out equal areas in equal time
    - The important implication is that planets move at different speeds during orbit
    - Faster when closest to Sun (perihelion), slower when farthest (aphelion)
  - 3rd Law – The square of the period is proportional to the cube of the average distance
    - That means if you graph T^2 vs r^3 for all the planets, they will form a line!
    - Use the semi-major axis like a radius for all the formulas in this section
- Newton's Law of Universal Gravitation
  - Force of attraction between two objects with mass
  - Constant **G = 6.67 x 10^{-11} N·m^2/kg^2 ≠ 9.8 m/s^2!**

$$\boldsymbol{F_g = G\frac{mM}{r^2}}$$

Here are some examples:

Compound Body Moving in Two Dimensions

Solve for acceleration: Fig 5.4a shows the scenario. As there are two masses there are two FBD's shown in Fig 5.4b. Fig 5.4c is an informal sketch of connected boxes. Use this sketch and the combined mass method to solve for overall acceleration. ***Remember, when you use the combined method you must total all the masses for the sum of force.*** F_g and F_N acting on mass A are perpendicular to motion. They cancel each other. Fig 5.4c shows that the ***tension in the rope also cancels***. It is the same rope so the value at both ends is the same and the direction of tension is opposite. So tension cancels in the shortcut to find acceleration.

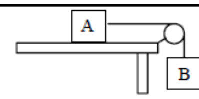


Fig 5.4a

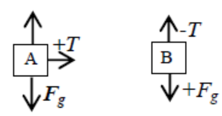


Fig 5.4b

$\Sigma F_{AB} = F_{gB}$ $\qquad (m_A + m_B)a = m_B g$ $\qquad \boxed{a = \dfrac{m_B g}{(m_A + m_B)}}$

Solve for the tension in the rope: In order to solve for tension you need a formula with tension in it. You must solve for one of the masses by itself. Solve for either body. On tests choose the easy one, this will usually be the hanging mass.

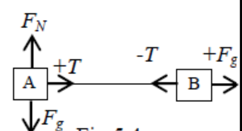


Fig 5.4c

$\Sigma F_A = T - F_{frA}$ or $\Sigma F_B = F_{gB} - T$

$\boxed{T = m_A a + \mu m_A g}$ $\qquad \boxed{T = m_B g - m_B a}$

Plug the acceleration from part one into either equation above and you will get the same final answer.

Atwood Machine: Atwood created a device to artificially slow the acceleration of gravity. In Fig 5.4a it doesn't say which mass is greater. I picked the two masses B and C as the more massive side and used this to set the direction of motion.

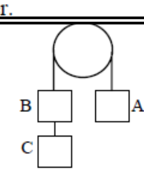


Fig 5.5a

Solve for acceleration: The FBD's for all blocks are shown in Fig 5.5b. Use the combined mass method to solve for overall acceleration. *Remember, when you use the combined method you must total all the masses for the sum of force.* In addition it is easier if you treat blocks B and C as though they are one larger block having a single mass. Fig 5.5c is a sketch of the masses as a linear problem, with the left masses combined.

$\Sigma F_{ABC} = F_{gBC} - F_{gA}$

$(m_A + m_B + m_C)a = (m_B + m_C)g - m_A g$ $\qquad a = \dfrac{(m_B + m_C)g - m_A g}{(m_A + m_B + m_C)}$

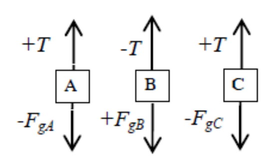


Fig 5.5b

If asked for the tension in the rope connecting mass A and B you must sum the forces for any block connected to the rope. A problem might only give information for one of the two blocks, or one of the blocks will be much simpler to solve. Learn to identify the easy block. If more than one mass is suspended by a rope, then add the masses suspended by the rope. This is the case for blocks B and C. Both of the possible solutions are detailed, one using block A on the left, and the other using blocks B and C to the right.

$\Sigma F_A = T - F_{gA}$ or $\Sigma F_{BC} = F_{gBC} - T$

$T = \Sigma F_A + F_{gA}$ $\qquad T = F_{gBC} - \Sigma F_{BC}$

$\boxed{T = m_A a + m_A g}$ $\qquad \boxed{T = (m_B + m_C)g - (m_B + m_C)a}$

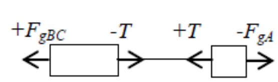
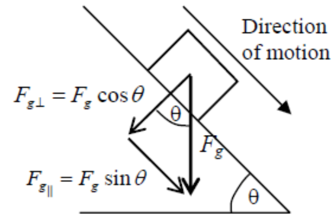


Fig 5.5c

Plug in the acceleration from above to solve for F_T. To solve for the tension in the rope between block B and C, just use the mass of block C. Refer to the right most FBD in Fig 5.5b.

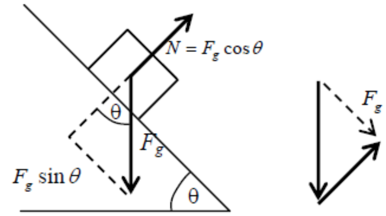
$\Sigma F_C = F_{gC} - T$ $\qquad T = F_{gC} - \Sigma F_C$ $\qquad \boxed{T = m_C g - m_C a}$

Force Gravity on Slopes: Motion on a slope is parallel to the slope. F_g is then at an angle to this motion. Any vector at an angle to motion should be split into components parallel and perpendicular to the chosen orientation (direction of motion). In this case F_g should split into $F_{g\parallel}$ and $F_{g\perp}$ to the slope. F_g is actually pulling the block into the slope $F_{g\perp} = F_g \cos\theta$ and down the slope $F_{g_\parallel} = F_g \sin\theta$.



But the block is not moving perpendicular to the slope, so the sum of forces in the perpendicular direction is zero. If gravity is pulling the block into the slope then the slope must push back with an equal and opposite force. Any force involving contact with a surface is call a **Normal Force,** N. In slope problems $\boxed{N = F_g \cos\theta}$.

When a block moves on a flat surface the slope angle is 0°. Plug 0° into the N equation and it reduces to $\boxed{N = F_g}$. This is an abbreviation for N that works only on flat surfaces. The component of gravity parallel to the slope $\boxed{F_{g\parallel} = F_g \sin\theta}$ is then left over. $F_{g\parallel}$ is the resultant when F_g and N are added by vector addition. This is the force of gravity that can accelerate objects down slopes.



Two Dimension Compound Body with Friction

Solve for acceleration: This is a repeat of Example 5-4, only this time friction appears in the FBD for mass A. Friction opposes motion and is therefore negative.

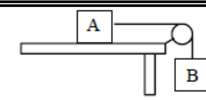


Fig 5.7a

$$\Sigma F_{AB} = F_{gB} - F_{frA} \qquad (m_A + m_B)a = m_B g - \mu m_A g \qquad \boxed{a = \frac{m_B g - \mu m_A g}{(m_A + m_B)}}$$

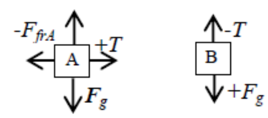


Fig 5.7b

Solve for the tension in the rope: Solve for either body. On tests choose the easy one, this will usually be the hanging mass.

$$\Sigma F_A = T - F_{frA} \qquad \text{or} \qquad \Sigma F_B = F_{gB} - T$$

$$\boxed{T = m_A a + \mu m_A g} \qquad \boxed{T = m_B g - m_B a}$$

Plug the acceleration from part one into either equation above and you will get the same final answer.

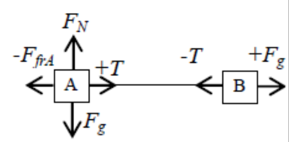


Fig 5.7c

Vertical Circular Motion

A ball at the end of a string is swung in a vertical circle. Any forces pointing to the center are positive, while force vectors pointing away from the center are negative. Sum the forces. In circular motion F_c is the sum of force. **Find the tension in the string when the ball is at the top and at the bottom.**

$$F_c = F_g + T_T \qquad F_c = -F_g + T_B$$

$$T_T = F_c - F_g \qquad T_B = F_c + F_g$$

$$\boxed{T_T = m\frac{v^2}{r} - mg} \qquad \boxed{T_B = m\frac{v^2}{r} + mg}$$

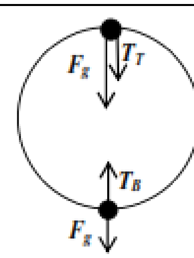


Fig 9.1

Horizontal Circular Motion

A penny on a circular disk rotating horizontally (or a car turning a corner). Something must be keeping it going in a circle. Friction keeps it in place. If friction let go the penny would move due to inertia in a direction tangent to the disk. Force centripetal is the sum of forces for circular motion. **Find a formula for the maximum velocity if the coefficient of friction is known.**

Fig 9.2

$$F_c = F_{fr} \qquad \cancel{m}\frac{v^2}{r} = \mu \cancel{m} g \qquad \boxed{v = \sqrt{\mu g r}}$$

Top of a Loop and Apparent Weightlessness

Apparently weightless means that you are in freefall. The only force acting on you is F_g. To feel weightless at the top of the loop the roller coaster car can have no F_N (no pressure from the track). So for an instant at the top the car is not touching the track. **What at the top of the loop makes this possible?**

$$F_c = F_g \qquad \cancel{m}\frac{v^2}{r} = \cancel{m}g$$

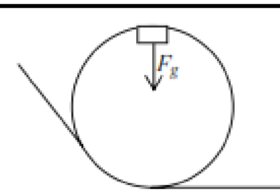


Fig 9.3

Unit 3: Work, Energy, and Power

- Work, energy and power are all scalar quantities
- Work (Joules)
  - When a force is exerted, making an object move, then work is done
  - Only the component of the force in the direction of movement does work
  - $\boldsymbol{W = Fdcos\theta}$ if **θ** the angle is between force and displacement
  - *Work is the area under a Force – Time curve*
- Kinetic Energy (Joules)
  - Energy of moving object $\boldsymbol{K = \frac{1}{2}mv^2}$
- Work – (Change in) Kinetic Energy
  - Derived from kinematics
  - $\boldsymbol{W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$
  - *This relationship is useful*
  - If W > 0, object is speeding up
  - If W < 0, object is slowing down
  - If W = 0. object moving at constant speed
- Gravitational Potential Energy (Joules)
  - Energy related to position in a gravitational field $\boldsymbol{U_g = mgh}$
  - Work done by the field is opposite the change in potential energy $\boldsymbol{W = -\Delta U}$
- Elastic Potential Energy (Joules)
  - Energy stored in elastic systems $\boldsymbol{U_{sp} = \frac{1}{2}kx^2}$
- Conservation of Energy
  - Mechanical Energy (no friction) $\boldsymbol{K_i + U_i = K_f + U_f}$
  - If object starts at rest and ends at zero level $\boldsymbol{v = \sqrt{2gh}}$
  - Work done by non-conservative force $\boldsymbol{W_{nc} = \Delta K + \Delta U = (K_f + U_f) - (K_i + U_i)}$
- Average Power (Watts)
  - Rate at which work or energy is done $\boldsymbol{P_{avg} = \frac{W}{\Delta t}}$ $\boldsymbol{P_{avg} = Fv}$

Example 6-4: Roller Coaster

A roller coaster is a good example of incomplete transfer of energy. At different points along the track there are various amounts of both kinetic and potential energy. One key to all these height problems, measured from a planetary surface, is to declare the lowest point in a problem to be $h = 0$. This gives a reference that is easy to add or subtract from. Then if you are given the height at any point on the track you can find the carts velocity:

$$\boxed{mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2}$$

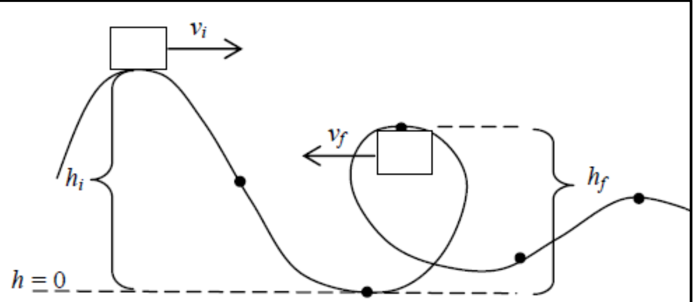


Fig 6.4

Unit 4: Linear Momentum

- Linear momentum (kg·m/s) is a vector
  - Depends on an object's mass and velocity $\boldsymbol{p = mv}$
  - It's really important to *make one direction positive and the other negative* for objects moving in opposite directions – like in head on collisions
- Impulse (kg·m/s) is a vector
  - Has to do with the time a force is applied to an object, causing a change in momentum
  - Can be found as the area under a force-time graph
  - $\boldsymbol{J = \Sigma F \Delta t = \Delta p = mv_f = mv_i}$
  - If you increase the time for an object to slow down, you decrease the force on object
- Conservation of momentum
  - Works when no external forces act on system $\boldsymbol{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$
- Collisions
  - Recoil – both objects begin together at rest and move apart $\boldsymbol{m_1 v_{1f} = -m_2 v_{2f}}$
  - Perfectly Inelastic – objects collide and stick together $\boldsymbol{m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f}$
  - Elastic – objects collide & separate w/ *KE conserved* $\boldsymbol{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$
  - Most real-life collisions are neither elastic nor perfectly inelastic, but simply inelastic
  - For collisions at an angle (glancing), use components and handle each axis separately

Example 8-1: Conservation of Linear Momentum

Elastic Collision: Mass 1, $m_1 = 2.00$ kg, is moving at 4.00 m/s to the right. Mass 2, $m_2 = 2.00$ kg, is stationary and is hit by mass 1. After the collision mass 2 moves to the right at 4.00 m/s. The collision is diagrammed before and after in Fig 8.1a. What is the velocity of mass 1 after the collision?

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(2)(4) + (2)(0) = (2)v_{1f} + (2)(4) \qquad \boxed{v_{1f} = 0 m/s}$$

All the momentum from mass 1 was transferred to mass 2.

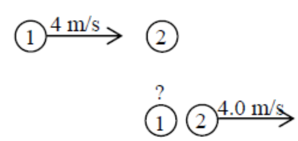


Fig 8.1a

Inelastic Collision: Mass 1, $m_1 = 4.00$ kg, is moving at 4.00 m/s to the right. Mass 2, $m_2 = 2.00$ kg, is stationary and is hit by mass 1. After the collision mass 2 moves to the right at 5.30 m/s. The collision is diagrammed before and after in Fig 8.1b. What is the velocity of mass 1 after the collision?

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(4)(4.00) + (2)(0) = (4)v_{1f} + (1)(5.30) \qquad \boxed{v_{1f} = 1.35 m/s}$$

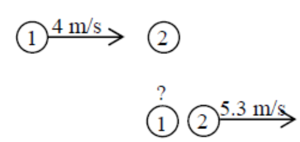


Fig 8.1b

Perfectly Inelastic Collision: Mass 1, $m_1 = 4.00$ kg, is moving at 4.00 m/s to the right. Mass 2, $m_2 = 2.00$ kg, is stationary and is hit by mass 1. After the collision the mass stick together. The collision is diagrammed before and after in Fig 8.1c. What is the velocity of the combined mass after the collision?

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$(4)(4.00) + (2)(0) = (4 + 2) v_f \qquad \boxed{v_{1f} = 2.67 m/s}$$

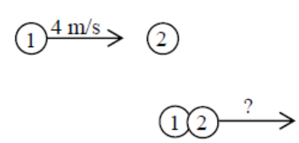


Fig 8.1c

Explosion: A large mass fractures into two smaller masses, $m_1 = 4.00$ kg and $m_2 = 2.00$ kg. Mass 2 moves to the right at 2.0 m/s. How fast is mass 1 going after the explosion?

$$(m_1 + m_2) v_i = m_1 v_{1f} + m_2 v_{2f}$$

$$(4 + 2)(0) = (4)v_{1f} + (2)(2) \qquad \boxed{v_{1f} = -1 m/s}$$

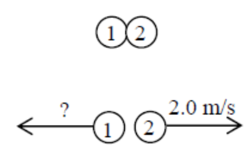


Fig 8.1d

The minus sign in the last example means that the mass is going to the left. Remember to watch the minus signs. Harder problems will have masses moving in different directions. Missing the sign convention will destroy the problem.

Unit 5: Torque and Rotational Dynamics

- Different points on a rotating object have different tangential displacements, velocities and accelerations, but the same angular displacements, velocities and accelerations. The equations below are vitally important to linking the worlds of linear and rotational motion together. Every rotational variable has a corresponding linear one. Learn the relationships and cut the number of formulas you need to memorize in half!
- Angular displacement, speed and acceleration — Linear to Rotational Motion
  - Angular displacement (radians) $\boldsymbol{\Delta\theta = \theta_f - \theta_i}$ $\boldsymbol{\Delta x = \Delta\theta \cdot r}$
  - Angular speed (rad/s) $\boldsymbol{\omega = \frac{\Delta\theta}{\Delta t}}$ $\boldsymbol{v = \Delta\omega \cdot r}$
  - Angular acceleration (rad/s^2) $\boldsymbol{\alpha = \frac{\Delta\omega}{\Delta t}}$ $\boldsymbol{a = \Delta\alpha \cdot r}$
- Center of mass is the point where an object will balance
  - Can be found by hanging object from two spots, drawing vertical lines from the point of attachment, and looking for an intersection. Or balancing it on a fulcrum (pivot point).
  - Can be found mathematically by summing the product of a mass and its position on an axis, then dividing by the total mass $\boldsymbol{x_{com} = \frac{\sum x \cdot m}{M}}$
- Moment of Inertia is a measure of how hard or easy it is to change the rotation of an object
  - It's all about ***mass distribution***
  - Unit is kg·m^2
  - The closer the mass is to the axis of rotation, the lower the moment
  - The farther out the mass is from the axis, the higher the moment
  - For point masses, $\boldsymbol{I = m \cdot r^2}$
  - $I_{hoop} = MR^2 > I_{disc} = \frac{1}{2}MR^2 > I_{sphere} = \frac{2}{5}MR^2$
  - If a hoop, disc and sphere have the same *M*ass and *R*adius, and are released from rest at the top of a ramp, the sphere makes it to the bottom first because of its lower moment
- Torque is the ability of a force to cause rotation
  - In rotation problems we look at the sum of torque (not the sum of force).
  - $\boldsymbol{\tau = Fr\sin\theta}$ where θ is the angle between the *F*orce and the *r*adius
  - Some students are not careful about the angle, so use the **perpendicular** component of the *F*orce or *r*adius (*sin* or *cos*) no matter the angle given $\boldsymbol{\tau = F_\perp \cdot r = F \cdot r_\perp}$
  - Strongest when the force is perpendicular to the lever arm (since sin 90^o = 1).
  - Equilibrium means all forces and torques are balanced $\boldsymbol{\sum F_x = 0}$ $\boldsymbol{\sum F_y = 0}$ $\boldsymbol{\sum \tau = 0}$
  - If clockwise and counterclockwise torques do not balance, rotation is accelerated
    - For Net Force we did "Forward – Backward" – it's the same thing here.
    - $\boldsymbol{F = ma}$ is Newton's 2nd Law so $\boldsymbol{\tau = I\alpha}$ is Newton's 2nd Law of Rotational Motion
  - Many questions will have you use both formulas for Torque, then substitute $\boldsymbol{\alpha = \frac{a}{r}}$

Torque and a Seesaw

Three masses are positioned on a seesaw as shown in Fig 10.1. $m_A = 4.0$ kg, $m_B = 2.0$ kg, and $m_3 = 3.0$ kg. Distances are shown in the diagram.

How far from the fulcrum must m_B be positioned in order for the system to balance? Keep in mind that measurements are made from the center of mass. It is as though all the mass is mathematically located at a point at the center of the object. It is not rotating, so the clockwise torques must equal the counterclockwise torques.

$\sum \tau_{cw} = \sum \tau_{ccw}$

$\tau_A = \tau_B + \tau_C$

$r_A \cdot F_A = r_B \cdot F_B + r_C \cdot F_C$

$r_A \cdot m_A \cancel{g} = r_B \cdot m_B \cancel{g} + r_C \cdot m_C \cancel{g}$

$(2)(4) = r_B(2) + (2)(3)$ $r_B = \boxed{1.0m}$

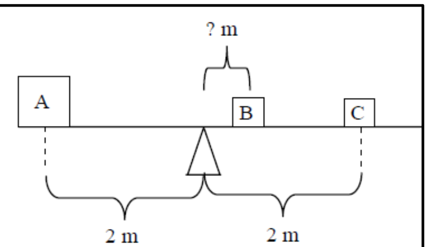


Fig 10.1

Unit 6: Energy and Momentum in Rotating Systems

- Rotational Kinetic Energy is a lot like Translational Kinetic Energy, but for rotating objects
  - ***$K_{translational} = \frac{1}{2} mv^2$*** and ***$K_{rotational} = \frac{1}{2} I\omega^2$***
  - Use this in Conservation of Energy problems and Rolling problems

Rolling Down an Incline

A spherical mass rolls 2 m down an incline shown in Fig 11.3a. The FBD for the sphere is shown in Fig 11.3b. In figure 11.3c the gravity and normal force vectors have been summed and the component of force down the slope is shown. $F_g\sin\theta$ pulls the sphere down the incline in the usual manner. However, the friction force vector is a distance R from the center of the sphere. These two vectors are perpendicular. This creates an unbalanced torque on the sphere, which causes it to rotate. The combined motion of rotating and moving down the slope is rolling.

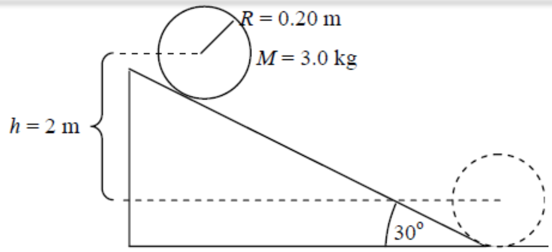
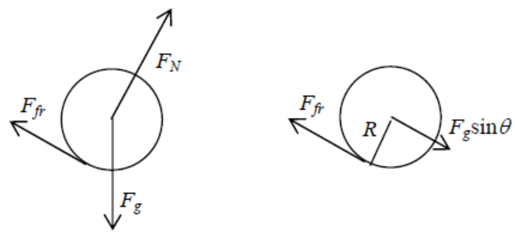


Fig 11.3a

What is the kinetic energy of the sphere at the bottom of the incline? The sphere is translating and rotating at the same time. The total kinetic energy is the addition of the translational and rotational kinetic energies.



$$K_{total} = K_{translation} + K_{rotation}$$

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$K_{total} = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 \qquad K_{total} = \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 \qquad K_{total} = \frac{7}{10}Mv^2$$

Rearrange for velocity, plug in values, and solve.

$$v = \sqrt{\frac{10}{7}\frac{K_{total}}{M}} \qquad v = \sqrt{\frac{10}{7}\left(\frac{58.8}{3.0}\right)} = \boxed{5.29\,m/s}$$

Another expression can be derive here also $v = \sqrt{\frac{10}{7}gh}$

- Angular Momentum ($kg{\cdot}m/s^2$) is a lot like linear momentum, but for rotating objects
  - $\boldsymbol{p = mv}$ is linear momentum and is $\boldsymbol{L = I\omega}$ angular momentum
  - Conserved in the absence of external torque (like friction or gravity)
  - $\boldsymbol{L_i = L_f}$
  - $\boldsymbol{I_i\omega_i = I_f\omega_f}$
  - This means as I gets smaller, ω gets bigger and the object rotates faster, and vice versa
    - An ice skater starts spinning with her arms out, then pulls arms in and spins faster
- Gravitational Potential Energy (J)
  - The earlier equation, ***$U_g = mgh$***, is useful <u>only near surface of the Earth</u>
  - For larger distances from the surface, use this formula: $\boldsymbol{U_g = -G\frac{mM}{r}}$
    - Notice that it's *r*, not *r^2*. The negative sign is vitally important!
    - You can use this with Conservation of Energy because the total mechanical energy of this system is constant – so, ***E = K + U*** (but *U* has a negative sign!)
- Orbital speed refers to the speed necessary to move in a circular orbit around a massive object
  - One way to do it is to divide the distance around (circumference) by the time it takes (period)
  - $\boldsymbol{v = \frac{2\pi r}{T}}$ Everyone forgets this important relationship!
  - But, since the gravitational force is directed towards the center of circular orbit, it's also a centripetal force!
  - The mass that matters is the <u>*central mass only*</u>
  - If you put these two velocity equations together and solve for period squared, T^2, you get Kepler's 3rd Law!

$$\boldsymbol{F_c = F_g}$$

$$\boldsymbol{\frac{mv^2}{r} = G\frac{mM}{r^2}}$$

$$\boldsymbol{v = \sqrt{G\frac{M}{r}}}$$

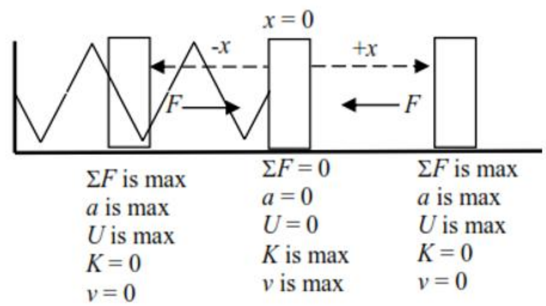
$$\boldsymbol{T^2 = \frac{4\pi^2}{GM}r^3}$$

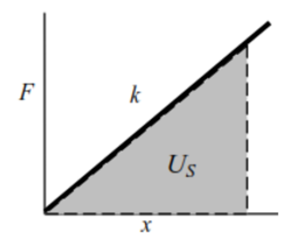
Once again, every angular variable has a linear counterpart. This table shows that the better you understand how to move from one type to the other, the better you will do and the fewer formulas you will have to know.

	Angular	**Linear**
Position	$\theta = \frac{arc\ length}{r}$	x
Displacement	$\Delta\theta = \theta - \theta_0$	$\Delta x = x - x_0$
Average Speed	$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$	$\bar{v} = \frac{\Delta x}{\Delta t}$ $\quad \bar{v} = \frac{v_0 + v}{2}$
Instantaneous Speed	$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$ $\quad \omega = \frac{d\theta}{dt}$	$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$ $\quad v = \frac{dx}{dt}$ **Slope of displacement - time graph**
Average Acceleration	$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$	$\bar{a} = \frac{\Delta v}{\Delta t}$
Instantaneous Acceleration	**Tangential Acceleration** $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$ $\quad \alpha = \frac{d\omega}{dt}$	$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$ $\quad a = \frac{dv}{dt}$ **Slope of velocity - time graph**
Kinematic Equations	$\boxed{\omega = \omega_0 + \alpha t}$ $\boxed{\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2}$ $\boxed{\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)}$	$\boxed{v = v_0 + at}$ $\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$ $\boxed{v^2 = v_0{}^2 + 2a(x - x_0)}$
Tangential Speed	$\boxed{v = r\omega}$ $\quad \omega = \frac{2\pi}{T}$	$v = \frac{2\pi r}{T}$
Centripetal Acceleration	**Radial Acceleration** $\boxed{a_c = \frac{v^2}{r} = \omega^2 r}$ Radial Acceleration is the acceleration directed along a radial (spoke) line. It is directed toward the center.	$a_c = \frac{v^2}{r}$
Inertia	**Moment of Inertia:** Depends on mass and distribution and thus varies for each object $\boxed{I = \int r^2 dm = \sum mr^2}$	m
Force and Torque	**Torque:** Unbalance torques cause rotation. $\boxed{\tau = \mathbf{r} \times \mathbf{F}}$ $\boxed{\sum \tau = \tau_{net} = I\alpha}$	**Force:** Unbalanced forces cause translation. $\mathbf{F}$ $\boxed{\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}}$
Kinetic Energy	$\boxed{K = \frac{1}{2}I\omega^2}$	$\boxed{K = \frac{1}{2}mv^2}$
Momentum	$\boxed{\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}}$	$\boxed{p = mv}$
Conservation of Momentum	$\mathbf{L}_i = \mathbf{L}_f$ $I\boldsymbol{\omega}_i = I\boldsymbol{\omega}_f$	$p_i = p_f$ $mv_i = mv_f$

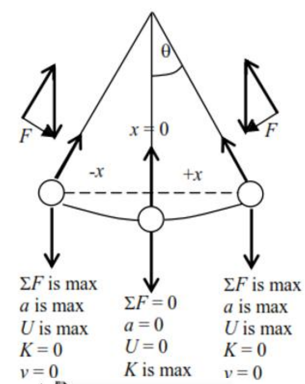
Unit 7: Oscillations

- SHM requires a restoring force (towards equilibrium) directly proportional to displacement
- Period is the time for one revolution (seconds) $\boldsymbol{T = \frac{1}{f}}$
- Frequency is number of revolutions, vibrations, oscillations, or rotations per second (Hz = s^{-1})
- Amplitude is distance from equilibrium to max displacement, not max displacement to other
- Springs
  - Spring force is a restoring force (***F = −kx***) is known as Hooke's Law
  - Spring constant (*k*) reflects the quality or strength of the spring
  - The minus sign shows that restoring force is opposite displacement
  - When object is at max displacement
    - spring force, acc and potential energy are at their greatest
    - velocity and kinetic energy are zero
  - When object is at equilibrium (*x* = 0),
    - velocity and kinetic energy are greatest
    - spring force, acceleration and potential energy are zero
  - If spring force is graphed against displacement, the slope is the spring constant (***F = kx***) and the area under the line is the spring potential energy (***Area = ½ bh = ½ Fx = ½ kx·x = ½ kx²***)

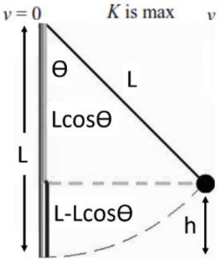




- Energy
  - Max potential energy ($\boldsymbol{U_s = \frac{1}{2} kx^2}$) at max displacement, zero at equilibrium
  - Max kinetic energy ($\boldsymbol{K = \frac{1}{2} mv^2}$) at equilibrium, zero at max displacement
- Period
  - Time for one oscillation depends on mass and k, NOT displacement! $\boldsymbol{T = 2\pi\sqrt{\frac{m}{k}}}$
- Pendulums
  - *mgsinΘ* is the restoring force
  - When mass is at max displacement
    - restoring force, acc and potential energy are at their greatest
    - velocity and kinetic energy are zero
  - When mass is at equilibrium (*x* = 0),
    - velocity and kinetic energy are greatest
    - restoring force, acceleration and potential energy are zero



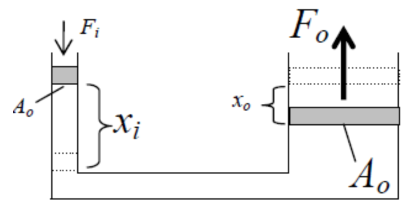
- Small Angle Approximation
  - Dashed displacement line is not the same length as the pendulum arc
  - The difference is very small at angles less than or equal to 15°
  - At angles under 15° pendulums behave approximately like a true SHM



- Energy
  - Max potential energy ($\boldsymbol{U_g = mgh}$) at max displacement
  - For a pendulum of length, *L*, with initial angle of *Θ*, ***h = L − LcosΘ***
  - Max kinetic energy ($\boldsymbol{K = \frac{1}{2} mv^2}$) at equilibrium
- Period
  - Time for oscillation depends on length and gravity, NOT mass or angle! $\boldsymbol{T = 2\pi\sqrt{\frac{L}{g}}}$

Unit 8: Fluids

- Two formulas lead the way, density and pressure, $\boldsymbol{\rho} = \frac{m}{V} = \frac{mass\ (kg)}{Volume\ (m^3)}$ and $\boldsymbol{P} = \frac{F}{A} = \frac{Force\ (N)}{Area\ (m^2)}$
- In metric units, the density of water is not 1, but 1000 kg/m^3 (note: $\boldsymbol{\rho} \neq \boldsymbol{P}$!)
- The pressure in a fluid increases with depth, *h*, measured from the surface $\boldsymbol{P = P_0 + \rho g h}$
- Usually, P_0 is atmospheric pressure, roughly 100,000 Pa NOT zero! (1 Pascal = 1 N/m^2)
- Pascal's Principle says that the pressure everywhere in an enclosed fluid is the same everywhere in that fluid. This is used for hydraulics problems, like lifting cars.



  - Since Pressure is the same, $\boldsymbol{P} = \frac{F_1}{A_1} = \frac{F_2}{A_2}$
  - But the Work is the same too, so $\boldsymbol{W = F_1 \cdot \Delta x_1 = F_2 \cdot \Delta x_2}$
  - So small *F* over small A for a large *Δx* equals large *F* over large A for a small *Δx*
- Buoyancy has to do with the upward force exerted on an object by a fluid and can be described in two ways – both of which are equal
  - Archimedes Principal says that the buoyant force is equal to the weight of the displaced fluid – which is equal to the density of the *fluid* times the volume of the object *submerged* (under the surface) times acceleration of gravity, or $\boldsymbol{F_B = \rho_f V_{sub} g}$
  - The other way to think of it is as the net force exerted on an object by the fluid pressure, and since the pressure exerted on the bottom of the object is greater than the pressure exerted on the top (pressure increases further down in a fluid), the net force is upward.
- If the object is completely submerged, then the buoyancy force involves the entire volume of the object. But if it's partially submerged (floating) then the buoyancy force involves only the volume of the object below the surface (displaced volume).

Buoyancy and Density: Density tells whether it will sink or float. Fig 12.3 diagrams several masses in a fluid.

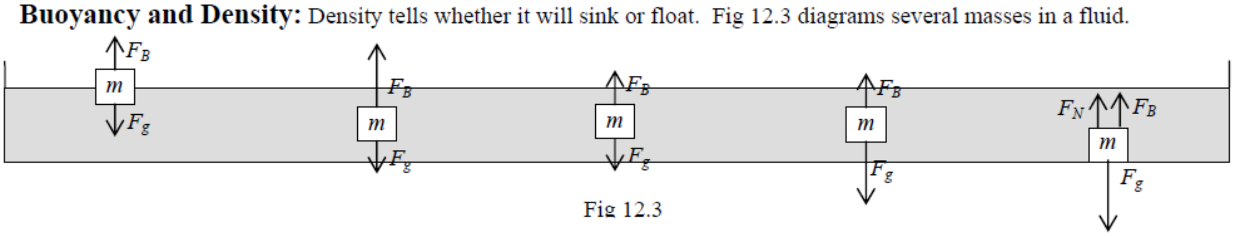


Fig 12.3

Floats out of water	Rising to Surface	Neutral Buoyancy	Sinking to Bottom	Resting on Bottom
$F_B = F_g$	$F_B > F_g$	$F_B = F_g$	$F_B < F_g$	$F_B + F_N = F_g$
$\rho_{obj} < \rho_{Fluid}$	$\rho_{obj} < \rho_{Fluid}$	$\rho_{obj} = \rho_{Fluid}$	$\rho_{obj} > \rho_{Fluid}$	$\rho_{obj} > \rho_{Fluid}$
$m_{obj} = m_{Fluid\ Displaced}$	$m_{obj} < m_{Fluid\ Displaced}$	$m_{obj} = m_{Fluid\ Displaced}$	$m_{obj} > m_{Fluid\ Displaced}$	$m_{obj} > m_{Fluid\ Displaced}$
$V_{obj} > V_{Fluid\ Displaced}$	$V_{obj} = V_{Fluid\ Displaced}$	$V_{obj} = V_{Fluid\ Displaced}$	$V_{obj} = V_{Fluid\ Displaced}$	$V_{obj} = V_{Fluid\ Displaced}$

Buoyancy

A 2.0 kg cube, that is 10.0 cm in length on each side, is suspended by both a gray fluid and by a spring as shown in Fig 12.4a. The spring constant is 100 N/m and the spring is stretched 0.15 m.

What is the density of the fluid? This is a balanced force problem. The object is not moving. Use the FBD in Fig 12.4b to sum the forces.

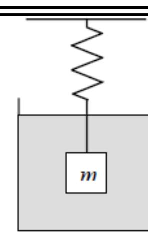


Fig 12.4a

$\sum F = F_S + F_B - F_g \qquad \sum F = 0$

$F_S + F_B = F_g$ Upward force spring and force buoyancy equals the downward force gravity.

$kx + \rho V g = mg$

$$\rho = \frac{mg - kx}{Vg} \qquad \rho = \frac{(2.0)(9.8) - (100)(0.15)}{(0.1)^3 (9.8)} = \boxed{469 \frac{kg}{m^3}}$$

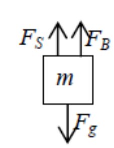


Fig 12.4b

- Flow Rate (m^3/s) = Area x Velocity represents the volume of fluid that passes a given point every second and is constant for ideal fluids.
    - That means that when a pipe narrows, the fluid velocity increases, and when a pipe widens, the fluid velocity decreases
    - This is called the Continuity Equation $\boldsymbol{Flow\ Rate = A_1 v_1 = A_2 v_2}$
- Bernoulli's Equation has to do with pressure, velocity and height of a fluid
    - $\boldsymbol{P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2}$
    - The P here represents pressure exerted on the fluid by outside forces. It is very often the case that $\boldsymbol{P_1 = P_2 = P_{atm}}$ and they cancel out, but be sure to show that step anyway
    - If the pipe is horizontal, then the $\boldsymbol{y}$ values cancel and what happens is that the faster a fluid flows, the lower the pressure will get. So in a horizontal pipe, if the pipe narrows, the fluid speeds up and the fluid pressure decreases. If it widens, the fluid slows down and the pressure of the fluid gets bigger.
    - Very often a Bernoulli Equation question will include a projectile portion, so be ready

A tube, shown in Fig 12.6a, that has an opening at the top and a hole in the side is continuously filled with water. It is filled so that the water level remains constant, even though water is leaking from the hole in the side.

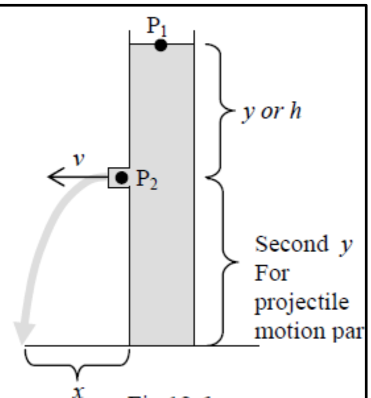


Fig 12.6a

What is the velocity of the water coming out of the hole in the side?

Use Bernoulli's Law $\boxed{p + \rho g y + \frac{1}{2}\rho v^2 = constant}$

Pick two convenient locations: The surface of the water (point 1) and the hole in the side (point 2). The fluid flows from the surface $\boxed{p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = constant}$ to the hole $p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 = constant$.

It is the same fluid, so the constant at both points must be the same. Therefore,

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

The tube is open to the atmosphere at both ends, so $p_1 = p_2$

$$\cancel{\rho} g y_1 + \frac{1}{2}\cancel{\rho} v_1^2 = \cancel{\rho} g y_2 + \frac{1}{2}\cancel{\rho} v_2^2$$

Since the level of the water is maintained at a constant point the velocity at point 1 is zero.

$g y_1 = g y_2 + \frac{1}{2}v_2^2$ Which simplifies to $v = \sqrt{2g(y_1 - y_2)}$ or $\boxed{v = \sqrt{2g\Delta y}}$