NOTE: TRIGONOMETRIC FUNCTIONS AND BASIC VECTOR MATHEMATICS HAVE NOT BEEN INCLUDED IN THIS SUMMARY.

Motion along a Straight Line.

I. $\bar{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta t}$, $\mathbf{v} = \frac{d\mathbf{x}}{dt}$

II. $\bar{\mathbf{a}} = \frac{\Delta \mathbf{v}}{\Delta t}$, $\mathbf{a} = \frac{d\mathbf{v}}{dt}$

For **constant a**:
I. $v = v_o + at$
II. $x = \frac{1}{2}at^2 + v_o t + x_o$
III. $v^2 = 2a\Delta x + v_o^2$

Motion in Two and Three Dimensions.

PROJECTILE MOTION: The equations of motion along a straight line applied to horizontal and vertical components.

CIRCULAR MOTION: I. $a_c = \frac{v^2}{r}$ directed *toward the center of the circle.*

$\therefore\ F_c = ma_c = \frac{mv^2}{r}$

note that an object may also experience a tangential force and acceleration while moving in a circle, then:

$a_T = \frac{dv_T}{dt}$ and $F_T = ma_T$

Force and Motion

Newton's Laws of Motion:
I. $\Sigma\ \mathbf{F} = 0 \leftrightarrow \mathbf{a} = 0$
II. $\mathbf{F} = m\mathbf{a}$ From this we have that weight = mg
III. $\mathbf{F}_{A,B} = -\mathbf{F}_{B,A}$

Friction:
I. $f \le \mu_s N$; static friction opposes motion with a force up to $\mu_s N$
II. $f = \mu_k N$

Work and Kinetic Energy.

I. $W = \int \mathbf{F} \cdot d\mathbf{s}$
II. $\mathbf{F} = -k\mathbf{d}$, Hooke's law.
III. The work done by a spring $W = \frac{1}{2}kx^2$
IV. $K = \frac{1}{2}mv^2$ The definition of kinetic energy.
V. The Work-Energy Theorem: The Work of the resultant force is equal to the change in kinetic energy. $W_R = \Delta K$
VI. $P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$

The Conservation of Energy.

I. $\Delta U = -W = -\int_{s_1}^{s_2} \mathbf{F}\cdot d\mathbf{s}$, here F must be a conservative force, i.e. a force such that the work done in moving from one place to another is the same regardless of the path, e.g. gravitation.

II. $U = mgh$, the gravitational potential energy *near* t he surface of the earth.

III. $F(x) = -\frac{dU}{dx}$ for a one dimensional conservative force.

IV. $U = \frac{1}{2}kx^2$, the elastic potential energy. (e.g. for a spring)

Systems of Particles.

I. Center of Mass: $x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$ $y_{cm} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}$ $z_{cm} = \frac{\sum_{i=1}^{n} m_i z_i}{\sum_{i=1}^{n} m_i}$

$x_{cm} = \frac{1}{M}\int x\,dm$ $y_{cm} = \frac{1}{M}\int y\,dm$ $z_{cm} = \frac{1}{M}\int z\,dm$

II. Newton's Second law for a system of particles: $\sum \mathbf{F}_{ext} = M\mathbf{a}_{cm}$

III. Momentum: $\mathbf{p} = m\mathbf{v}$

IV. Newton's actual expression for the second law: $\sum \mathbf{F} = \frac{d\mathbf{p}}{dt}$

V. *Momentum is conserved if no external force acts on a system.*

Collisions.

I. Impulse = $\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F}dt = \Delta\mathbf{p}$ this is actually an integral form for $\mathbf{F} = \frac{d\mathbf{p}}{dt}$, where impulse, $\mathbf{F}dt$, equals the change in momentum.

II. *Elastic Collisions* are those in which **Kinetic Energy** is conserved.

III. *If particles stick together as the result of a collision, they will have lost as much of their kinetic energy as the law of conservation of momentum will all. We call such collisions completely inelastic.*

Rotation.

I. $\theta = \frac{s}{r}$ II. $\omega = \frac{d\theta}{dt}$ III. $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

For **constant** α: IV. $\omega = \omega_o + \alpha t$

V. $\theta = \frac{1}{2}\alpha t^2 + \omega_o t + \theta_o$

VI. $\omega^2 = 2\alpha\Delta\theta + \omega_o^2$

VII. Moment of inertia: system of particles $I = \Sigma m_i r_i^2$

continuous mass distribution $I = \int r^2 dm$

parallel axis theroem $I = I_{cm} + Mh^2$

VIII. Kinetic Energy of ratation: $K = \frac{1}{2} I\omega^2$ IX. $\tau = \mathbf{r} \times \mathbf{F}$ X. $\Sigma\, \tau = I\alpha$

Rolling, Torque, and Angular Momentum

I. For Rolling without slipping: $v_{cm} = R\omega$

II. The Kinetic Energy of a rigid body undergoing both rotation and translation can be expressed as:

A) $K = K_{cm} + K_{rot} = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$, or

B) $K = \frac{1}{2} I_p \omega^2$, where I_p is the moment of inertia about a stationary axis parallel to the axis of rotation, e.g. an axis through the point of contact of a rolling object and a surface.

V. $\mathbf{l} = \mathbf{r} \times \mathbf{p}$ VI. $\mathbf{L} = \sum \mathbf{l}$ VII. $\tau = \frac{d\mathbf{L}}{dt}$

VIII. *Angular Momentum, **L** , is conserved if no external torques act on the object.*

IX. $L = I\omega$ component parallel to the axis for a rigid body rotating about a fixed axis

Equilibrium and Elasticity.

I. The two conditions for equilibrium are:

A) $\sum \mathbf{F} = 0 \leftrightarrow \mathbf{a} = 0$ B) $\sum \tau = 0 \leftrightarrow \alpha = 0$

N.B. it is only external forces and torques that can accelrate a system.

II. Elasticity:

A) In general: stress = modulus X strain

B) For objects undergoing compression or tension…

1. the stress is defined as Force/Area
2. the strain is defined as change in length/unit length
3. the modulus is called Young's Modulus (E).

$$\therefore \quad \frac{F}{A} = E \frac{\Delta L}{L}$$

Oscillations.

I. The Basic Condition for SHM: $\mathbf{a} = \frac{-k}{m} \mathbf{x}$ or $\frac{d^2\mathbf{x}}{dt^2} = \frac{-k}{m} \mathbf{x}$

II. The solution for the differential equation in I. is always of the form:

$x = A \sin(\omega t + \delta)$ *or the equivalent* $x = A \cos(\omega t + \delta)$ the difference being in the choice of δ. Remember that $v = \frac{dx}{dt}$, $a = \frac{dv}{dt}$, and $K = \frac{1}{2} mv^2$

III. $T = \frac{2\pi}{|\omega|}$, where T is the period.

IV. $f = \frac{1}{T}$, where f is the frequency. (sometimes ν,"nu", is used for frequency)

V. ω is determined as follows for different applications:

A) Spring ("massless") of constant k with a mass m attached…

$$\omega = \sqrt{\frac{k}{m}}$$

B) Simple Pendulum, mass on an end of a "massless" string…

$$\omega = \sqrt{\frac{g}{L}}$$

C) Torsion Pendulum, where $\tau = -\kappa\theta$, substitutes for $F = -kx$…

$$\omega = \sqrt{\frac{\kappa}{I}}$$

D) Physical Pendulum…

$$\omega = \sqrt{\frac{mgh}{I}}$$

where h is the distance from the cm to the pivot and I is taken about the pivot.

Gravitation.

I. Newton's law of gravitation: $F = G\frac{m_1 m_2}{r^2}$

This applies directly to "point masses and in the region outside of spherical masses. Calculus must be used to sum up the gravitation forces due to masses of other shapes.

II. $U = -G\frac{mM}{r}$, the potential energy "stored" in a two mas system due to their gravitational attraction. Masses must be points or spherical. The negative sign comes from assigning the value of potential energy at infinity as zero.

III. Motion of a Satellite/Planet: Combining the Law of Gravitation and the Second Law can yield the following basic equations…

A) Orbital Speed $v = \sqrt{\frac{GM}{r}}$ Where M is the mass of the object at the center of the orbit.

B) From A) the period of the orbit can be found from $T = \frac{2\pi r}{v}$

C) Escape speed near the surface of a body $v = \sqrt{\frac{2GM}{R}}$

IV. Kepler's Laws of Planetary Motion.

A) The Law of Orbits. All planets move in elliptical orbits with the sun at one focus.

B) The Law of Areas. A line joining any planet to the sun sweeps out equal areas in equal times.

C) The Law of Periods. The square of the period, T, of any planet about the sun is proportional to the cube of the semimajor axis of the orbit, i.e. $\frac{T^2}{r^3}$ = a constant.

These three laws can also be applied to orbiting systems other than the sun and its planets.