Electric Charge.

I. $F = k\dfrac{q_1 q_2}{r^2} = \dfrac{1}{4\pi\varepsilon_o}\dfrac{q_1 q_2}{r^2}$ for *point* charges and outside of a spherical charge distribution.

The vector form of the above equation in polar coordinates is… $\vec{F} = \dfrac{1}{4\pi\varepsilon_o}\dfrac{q_1 q_2}{r^2}\hat{r}$

The Electric Field.

I. $\mathbf{E} = \dfrac{\mathbf{F}}{q}$

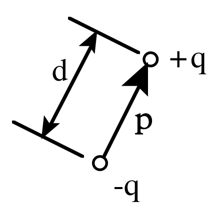
II. The electric field about a point charge or outside a spherical charge distribution.

$\mathbf{E} = \dfrac{1}{4\pi\varepsilon_o}\dfrac{q}{r^2}\,\hat{\mathbf{r}}$ N.B. There is no field inside a **conductor** having a static charge.

III. Some measures of electric charge…

charge of an object	q	in Coulombs
linear charge density	λ	in Coulombs/m
surface charge density	σ	in Coulombs/m^2
volume charge density	ρ	in Coulombs/m^3

IV. The vector dipole moment **p** is defined as having magnitude *qd* and a direction from the negative to the positive charge in the electric dipole.

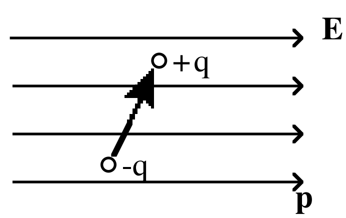


$p = qd$

V. Torque on an electric dipole placed in an electric field.

$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$

e.g.



here the torque would turn the dipole clockwise

VI. Potential Energy of a dipole.

$U(\theta) = -\,\mathbf{p}\cdot\mathbf{E}$ Here U is a minimum (most negative) when p is in the direction of the field.

Gauss's Law.

I. The Surface Vector d**A** is defined has having the magnitude, dA, of the surface increment under consideration and a direction *perpendicular to the plane* of the surface. If the surface is closed, then d**A** is taken as *outward*.

II. The flux of an electric field. $\Phi = \int \vec{E} \cdot d\vec{A}$ through a surface.

$\Phi = \oint \vec{E} \cdot d\vec{A}$ through a *closed* surface.

III. Gauss's Law. $\varepsilon_o \oint \vec{E} \cdot d\vec{A} = q$ where q is the **net** charge contained **within** the closed surface.

IV. Some electric fields commonly determined by reference to Gauss's Law.

a) Electric field caused by an infinite sheet of charge: $E = \dfrac{\sigma}{2\varepsilon_o}$

b) Electric field caused by large parallel plates of equal and opposite charge: $E = \dfrac{\sigma}{\varepsilon_o}$

c) Electric field of a long line of charge: $E = \dfrac{\lambda}{2\pi\varepsilon_o r}$ at a distance r from the charge

Electric Potential.

I. The equation for potential energy difference is, of course, the same as it was in mechanics…

$U_f - U_i = -\int_i^f \mathbf{F} \cdot d\mathbf{s}$ Where **F** is the force associated with the potential energy, e.g. the force of a spring, the gravitational attraction, or the electrical force of a field.

II. Potential Energy of a system of 2 pt. charges q and q'.

$$U = \frac{1}{4\pi\varepsilon_o}\frac{qq'}{r}$$ N.B. r is not squared.

III. Potential. $V = \dfrac{U}{q'}$ where V is in J/C or volts.

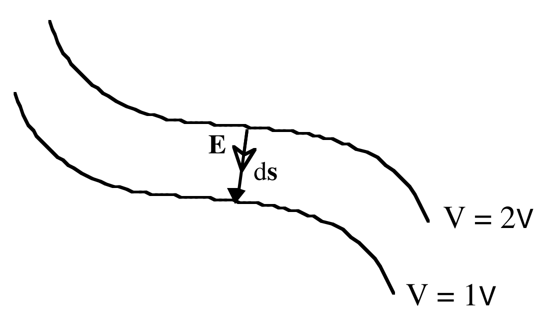
IV. Potential Difference. $V_f - V_i = -\int_i^f \mathbf{E} \cdot d\mathbf{s}$

This comes from dividing the potential energy equation by q.

V. Potential Gradient. *The rate of change of the potential with distance in any direction is, when changed in sign, the component of* ***E*** *in that direction.*

$$E_s = -\frac{dV}{ds}$$

Note that the field is perpendicular to the equipotential lines and points in the direction of the decrease in potential, i.e. it points down the potential hill.

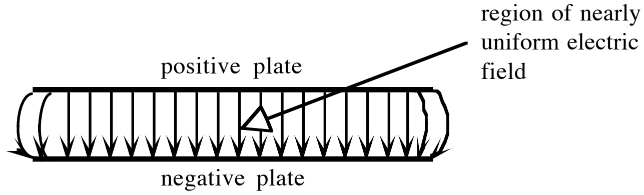


Capacitance.

I. Capacitance: $C = \frac{Q}{V}$, where C is in coulombs/volt or Farads (F)

The μF,nF, and pF are more practical expressions for capacitance.

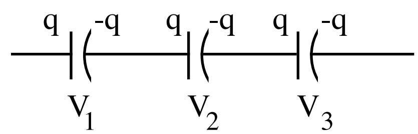
II. Parallel conducting plates with equal and opposite charges.



The field intensity, E, between the plates is considered uniform with negligible fringing at the edges, the difference in potential between the plates is given by $\Delta V = Ed$.

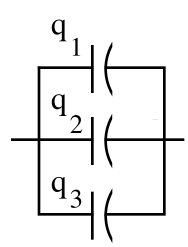
III. Parallel Plate Capacitor: $C = \varepsilon_o \frac{A}{d}$

IV. Capacitors in Series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n} = \sum_{j=1}^{n} \frac{1}{C_j}$



In this type connection, + to -, the *size of the charge* on all plates *is the same*, and the total **potential difference** is the **sum** of the individual potential differences.

V. Capacitors in Parallel: $C_{eq} = C_1 + C_2 + \cdots + C_n = \sum_{j=1}^{n} C_j$



In this type connection, + to + and - to -, the *the sum of the charges* of each capacitor equals the *total stored charge*, and the **potential difference** across each capacitor **is the same**.

VI. Energy of a charged capacitor: $U = \frac{1}{2} CV^2$

VII. Effect of a Dielectric: Dielectric Constant $\kappa = C/C_o$

e.g. the equation for a parallel plate capacitor becomes $C = \kappa\varepsilon_o \frac{A}{d}$

It can be shown that: $\kappa = C/C_o = V_o/V = E_o/E = \sigma/(\sigma - \sigma_i)$ where σ_i is the magnitude of the induced charge per unit area on the surface of the dielectric, and σ is the charge per unit area on the conducting plates.

VIII. With a dielectric present Gauss's Law becomes: $\varepsilon_o \oint \kappa \mathbf{E} \cdot d\mathbf{A} = q$

Current & Resistance.

I. Current: $i = \frac{dq}{dt}$ $\quad$ i in coulombs/sec or amperes (A).

II. Current Density: $J = \frac{i}{A}$ $\quad$ or more precisely $i = \oint \mathbf{J} \cdot d\mathbf{A}$

III. $\mathbf{J} = (ne)\mathbf{v}_d$

n = the number of charge carriers/unit volume.
e = the charge on each one
$\mathbf{v}_d$ = the average drift velocity

N.B. that if e is positive, that is if the charge carriers are positive, **J** is in the direction of their motion, and if e is negative, for example the electrons moving in a wire, **J** is in the direction opposite to the drift velocity. That is to say the <u>convention</u> is that current is in the direction of motion of positive charge carriers.

IV. Definition of Resistance: $R = \frac{V}{i}$ $\quad$ $\frac{1 \text{ volt}}{1 \text{ ampere}} = 1 \text{ ohm } (1\Omega)$

V. Definition of Resistivity: $\rho = \frac{E}{J}$ $\quad$ The units for ρ come out ohm· meters

VI. $R = \rho\frac{L}{A}$ $\quad$ for a homogeneous isotropic conductor of uniform cross sectional area,A, and length L.

VII. $V = IR$ $\quad$ For the case where R is a constant, this is Ohm's Law.

VIII. $P = \frac{dW}{dt} = iV = i^2R$ $\quad$ the resistive power dissipation in J/s or watts (W)

Circuits.

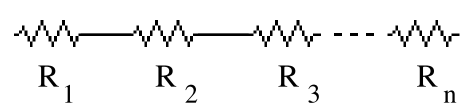
I. Definition of $\mathcal{E} = \frac{dW}{dq}$ $\quad$ This, the electromotive "force", is the energy per coulomb supplied to the circuit, where as V the potential difference is the drop in energy per coulomb over a section of a circuit.

II. $V = \mathcal{E} - ir$ Where V is the terminal voltage, $\mathcal{E}$ is the emf, and r is the internal resistance of the source. This could also be written as $\mathcal{E} = V + ir = iR + ir$, where it is evident that the energy supplied per coulomb is equal to the sum of the energy losses per coulomb in the external and internal circuits.

III. Kirchhoff's 1st Rule (**Junction Rule**): $\sum i_{in} = \sum i_{out}$ for any junction.

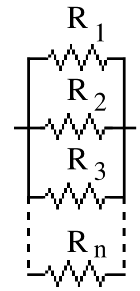
IV. Kirchhoff's 2nd Rule (**Loop Rule**): $\sum \Delta V = 0$ around any closed loop.

V. Resistances in Series: $R_{eq} = R_1 + R_2 + \cdots + R_n = \sum_{j=1}^{n} R_j$



In this type of connection the *<u>current is the same in all of the resistors</u>*, and the **total potential difference** across the resistors is the **sum of the potential drops across each**.

VI. Resistances in Parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} = \sum_{j=1}^{n} \frac{1}{R_j}$



In this type of connection the *<u>currents</u>* in each branch *<u>add up to the total current</u>* through the circuit, and the **potential difference** across each resistor **is the same**.

VII. The RC Series Circuit

<u>Charging Capacitor</u>	<u>Discharging Capacitor</u>
$q = C\mathcal{E}(1 - e^{-t/RC})$	$q = q_o e^{-t/RC}$
$i = i_o e^{-t/RC}$	$i = i_o e^{-t/RC}$

t = RC is the *capacitive time constant* or *relaxation time*.

For <u>charging</u>: when t = RC the charge has increased to **within** approximately 37% of its final value; the current has declined to about 37% of its initial value.
For <u>discharging</u>: when t = RC the charge **remaining** is 37% of its initial value and the current is 37% of its initial value.

The Magnetic Field.

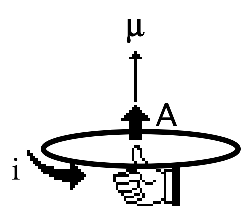
I. Force on a moving charge: $\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$

Where the unit of B is the tesla (T), and $1\text{T} = 1 \frac{\text{N}}{\text{A}\cdot\text{m}}$

II. Force on a current carrying conductor: $\mathbf{F} = i\mathbf{L} \times \mathbf{B}$ Here **L** is a vector that points along the wire segment in the direction of conventional current.

III. Magnetic Moment: $\mu = I\mathbf{A}$

Here we have a current loop.



The sense of A and therefore μ comes from the right hand rule. With the fingers of the right hand in the direction of the current the extended thumb will point in the sense of **A** and μ .

IV. Torque on a current loop: $\tau = \mu \times \mathbf{B}$

V. The Potential Energy of a Current Loop in a Magnetic Field: $U = -\mu \cdot \mathbf{B}$
Where U = 0 when μ is perpendicular to **B.** N.B. this is <u>not</u> a minimum <u>or</u> a maximum.

Ampere's Law.

I. $d\mathbf{B} = \frac{\mu_o}{4\pi}\frac{i d\mathbf{s} \times \mathbf{r}}{r^3}$ {The Biot-Savart Law}

Here μ_o is the *permeability constant* whose exact value, by definition, is $4\pi \times 10^{-7}$ T·m/A.

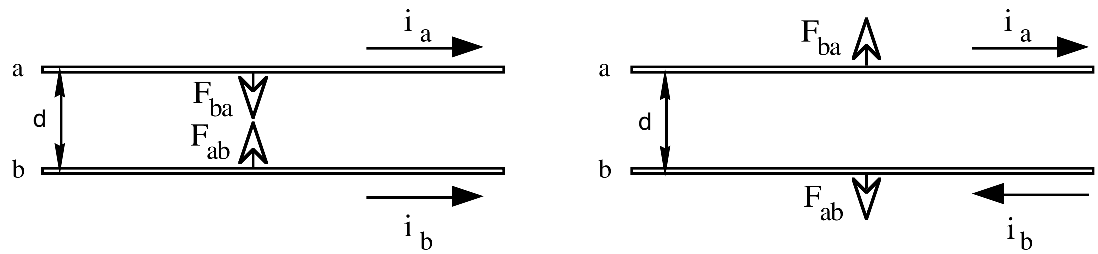
This could also be written as: $d\mathbf{B} = \frac{\mu_o}{4\pi}\frac{i d\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$

II. The magnetic field about a long straight wire: $B(r) = \frac{\mu_o i}{2\pi r}$

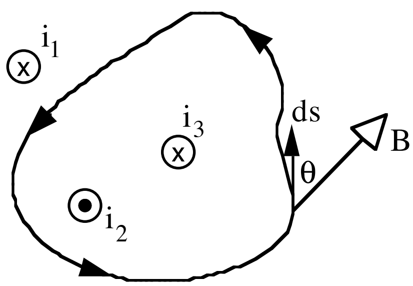
III. The magnetic field at the center of a circular current loop: $B = \frac{\mu_o i}{2a}$ For more than one loop multiply by N.

IV. The force between parallel current carrying wires: $F_{ab} = i_b L B_a = \frac{\mu_o L i_a i_b}{2\pi d}$

The force is attractive if the currents are in the direction and repulsive if the currents are in opposite directions.



V. Ampere's Law : $\oint \mathbf{B} \cdot d\mathbf{s} = \mu_o i$ i is the net current surrounded by the closed Amperian loop. {N.B. This is a *Line integral* not a surface integral like that encountered in Gauss's Law.}

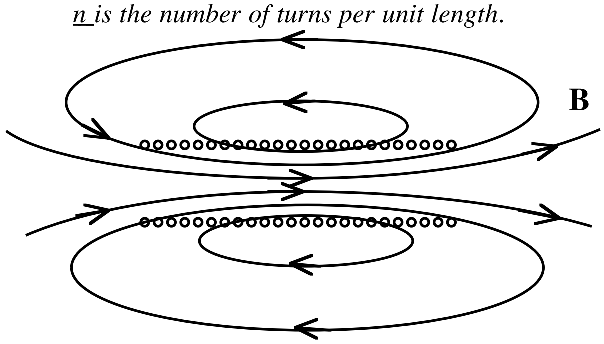


i net for the Amperian loop shown is $= |i_3 - i_2|$

VI. Some results easily Calculated from Ampere's Law.

a) The magnetic field near the center of a long current carrying solenoid: $B = \mu_o i_o n$

<u>*n*</u> *is the number of turns per unit length.*



A cross sectional view. Here the current is coming toward the observer on the top wires and going away in the bottom wires.

b) The magnetic field inside a toroid: $B = \frac{\mu_o i_o N}{2\pi r}$

<u>*N*</u> *is the total number of turns, and r is the radius of the loop of the magnetic field line. This last fact makes the field non uniform and strongest near the inside wall of the toroid.*

Faraday's Law of Induction.

I. Definition of Magnetic Flux: $\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$ This is similar to the definition of Electric Flux seen above.

The SI units are $T \cdot m^2$ named the *weber* (Wb).

II. Faraday's Law: $\mathcal{E} = -\frac{d\Phi_B}{dt}$ An *induced* emf is equal to the time rate of change of magnetic flux and in such a direction that it opposes that which causes the flux change.

III. Lenz's Law: "An induced current in a closed loop will appear in such a direction that it opposes the change which produced it."

IV. Faraday's Law (revisited): $\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$
{The integral is a *Line Integral* not the surface integral used above for Gauss's Law.} This formulation of Faraday's Law states that "the Line Integral of the Electric field around any closed loop is equal to the emf in the loop and therefore equal to the time rate of change of the magnetic flux through the loop". E is the non-electrostatic field. The negative sign is again indicative of the direction of the emf and is tied to Lenz's Law.

V. For a straight wire moving with a velocity **v** through a uniform magnetic field **B** the induced emf is given by: $\mathcal{E} = \mathbf{v} \times \mathbf{B} \cdot \mathbf{L}$ A common example is the case where the wire is perpendicular to the field and the velocity is perpendicular to both the field and the wire. In this case we have $\mathcal{E} = vBL$

Inductance.

I. Definition of Inductance: $L = \frac{N\Phi}{i}$ N is the number of turns linked by the magnetic flux Φ each turn carrying a current i.

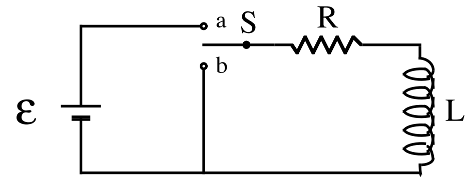
The SI units are $T \cdot m^2/A$ named the *henry* (H).

II. The inductance per unit length of a Solenoid: $\frac{L}{l} = \mu_o n^2 A$ Where n is the number of turns per unit length.

III. The inductance of a Toroid: $L = \frac{\mu_o N^2 h}{2\pi} \ln \frac{b}{a}$ Where a is the inner radius and b is the outer radius as measured from the center of the toroid, and h is the height.

IV. Self-Induced emf: $\mathcal{E} = -L \frac{di}{dt}$ The direction of the emf can be determined by Lenz's Law.

V. The LR Series Circuit: By using Kirchhoff's Loop rule for the circuit shown here, we get the set of equations and their solutions as given immediately below the diagram.



At t=0 switch put to a begins the rise of current.

After the current has been established, if the switch is movd to b, t=0, the decay of current will begin.

	Rise of Current	Decay of Current
Equation:	$iR + L\frac{di}{dt} = \varepsilon$	$iR + L\frac{di}{dt} = 0$
Solution:	$i = \frac{\varepsilon}{R}(1 - e^{-tR/L})$	$i = i_o e^{-tR/L}$

$t = \frac{L}{R}$ is the *time constant*.

Starting from an **open** circuit: $t = \frac{L}{R}$ represents the time to reach about 63% of the final current (or within about 37% of the maximum current). Compare this to RC circuits where t = RC.

Starting with a **closed** circuit: $t = \frac{L}{R}$ represents the time required for the current to drop to 37% of its initial, maximum, value.

VI. Energy *stored* in the magnetic field of an inductor: $U_E = \frac{1}{2} Li^2$

Here i is the "final" or peak current in the inductor.

VII. Mutual Inductance: $M_{21} = \frac{N_2\Phi_{21}}{i_1}$ The Mutual Inductance of coil 2 with respect to coil 1 caused by the current in coil one. Units are again the henry. It turns out that the mutual inductance of coil 1 with respect to coil 2 will come out the same number of henrys.

VIII. $\varepsilon_2 = - M \frac{di_1}{dt}$

Maxwell's Equations.

Maxwell developed four equations that are regarded as the basis for all electrical and magnetic phenomena. In fact, when these are applied to free space they the laws of Gauss, Faraday, and Ampere with the latter modified by Maxwell to include magnetic fields caused by a changing electrical flux. The equations are"

I. $\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_o}$ Gauss's Law

II. $\oint \vec{B} \cdot d\vec{A} = 0$ Gauss's Law for Magnetism

III. $\oint \vec{E} \cdot d\vec{S} = -\frac{d\phi_B}{dt}$ Faraday's Law

IV. $\oint \vec{B} \cdot d\vec{S} = \mu_0 I + \varepsilon_0\mu_0 \frac{d\phi_E}{dt}$ Ampere-Maxwell Law

Note that in the Ampere-Maxwell Law the induced magnetic field caused by an increasing electric flux is in the same direction as that cause by a current in the direction of the increase.